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Semiclassical strings in $AdS_3 \times S^2$

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ABSTRACT: In this paper, we investigate the semiclassical strings in $AdS_3 \times S^2$, in which the string configuration of AdS_3 is classified to three cases depending on the parameters. Each of these has a different anomalous dimension proportional to $\log S$, $S^{1/3}$ and S , where S is a angular momentum on AdS_3 . Further we generalize the dispersion relations for various string configuration on $AdS_3 \times S^2$.

KEYWORDS: Long strings, AdS-CFT Correspondence

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1 Introduction

A remarkable development in the study of string theory of last decade is the celebrated string theory-gauge theory duality [1], which relates the spectrum of semiclassical string states on $AdS_5 \times S^5$ to the operator dimensions of $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory in four dimensions [2, 3]. This state-operator matching has been achieved in the planar limit, which simplifies considerably both sides of the duality. Recently another example of gauge theory-string theory duality has been proposed based on the worldvolume dynamics of multiple M2-branes. This relates type IIA string theory in $AdS_4 \times CP^3$ with $\mathcal{N} = 6$ Chern-Simons matter theory in three dimensions [4].¹

In the study of the gauge/gravity duality, an interesting observation is that the $\mathcal{N} = 4$ SYM theory can be described by the integrable spin chain model [12–14]. It was further noticed that the string theory also has an integrable structure in the semiclassical limit. This integrability nature of both the gauge theory and string theory side has been used extensively to understand the duality better. In this connection, Hofman and Maldacena (HM) [15] considered a special limit where the problem of determining the spectrum of both sides becomes rather simple. The spectrum consists of an elementary excitation known as magnon which propagates with a conserved momentum p along the long spin chain. In the dual formulation, the most important ingredient is semiclassical string solutions, which can be mapped to long trace operators with large energy and large angular momenta. For some related work see for example [16]–[31].

Not so long back the giant magnon and spike solutions for the string on S^2 or S^3 have been studied [32–34]. In addition, the solitonic string configuration in the AdS space, whose

¹For related interesting work see [5–11].

anomalous dimension corresponds to that of the twist two operator of SYM theory or the cusp anomaly, was considered [35–38]. More recently the semiclassical string configuration on $AdS_3 \times S^3$ has also been investigated in a special parameter region [30, 39]. However a general class of solutions for string moving in $AdS_3 \times S^3$ background has been lacking. The aim of this paper is to classify such solutions with the hope that a better understanding of these will enable us to investigate more complicated solutions of the dual theory on the boundary.

In this paper, we consider the semiclassical strings moving in $AdS_3 \times S^2$ background and investigate the relation among various conserved charges of the string configuration at various parameter regions. Depending on the parameter region, the macroscopic strings on AdS_3 can be classified to three cases, whose anomalous dimension is proportional to $\log S$, $S^{1/3}$ or S , where S is a conserved angular momentum on AdS_3 . After that, we generalize rotating string on AdS_3 to the one rotating on $AdS_3 \times S^2$ and study the corresponding dispersion relation or the anomalous dimension.

The rest of the paper is organized as follows. In the section 2, we write down the most general equations describing the rotating string in $AdS_3 \times S^2$ background. In the section 3, first we classify various possible string configuration in AdS_3 . Depending on the region of parameter there are three distinct solutions with different anomalous dimensions. We further generalize them further by looking at the various shapes of the string. We write down the relevant dispersion relations in all the cases. Section 4 is devoted to the study of more general string configurations in $AdS_3 \times S^2$ and we find out the dispersion relations among various conserved charges. In section 5, we present our conclusions.

2 Semiclassical string configurations in $AdS_3 \times S^2$

In this section we will study the general rotating string solution in $AdS_3 \times S^2$. We start by writing down the relevant metric for $AdS_3 \times S^2$ in a particular coordinate system,

$$ds^2 = \frac{1}{4}R^2 [-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\phi^2 + d\theta^2 + \sin^2 \theta d\psi^2]. \quad (2.1)$$

The Polyakov action for the string moving in this background can be written as

$$S = \frac{1}{4\pi} \int d^2\sigma \sqrt{-\det h} h^{\alpha\beta} \partial_\alpha x^\mu \partial_\beta x^\nu G_{\mu\nu}. \quad (2.2)$$

We choose the following parametrization for the semiclassical rotating string in this background

$$\begin{aligned} t &= k\tau + h_1(y), & \rho &= \rho(y), & \phi &= \omega\tau + h_2(y), \\ \theta &= \theta(y), & \psi &= \nu\tau + g(y), \end{aligned} \quad (2.3)$$

where $y = a\tau + b\sigma$. The relevant equations of motion reduce to

$$\begin{aligned} h'_1 &= \frac{1}{b^2 - a^2} \left(ak - \frac{\pi_0}{\cosh^2 \rho} \right), \\ h'_2 &= \frac{1}{b^2 - a^2} \left(a\omega - \frac{\pi_2}{\sinh^2 \rho} \right), \\ g' &= \frac{1}{b^2 - a^2} \left(a\nu - \frac{\pi_4}{\sin^2 \theta} \right), \end{aligned} \quad (2.4)$$

where π_0 , π_2 and π_4 are integration constants. The above equations of motion have to be supplemented by the Virasoro constraints, which are simply given by, in a particular form

$$\begin{aligned} 0 &= T_{\tau\tau} + T_{\sigma\sigma} - \frac{a^2 + b^2}{ab} T_{\tau\sigma}, \\ 0 &= T_{\tau\tau} + T_{\sigma\sigma} - 2T_{\tau\sigma}. \end{aligned} \quad (2.5)$$

The first Virasoro constraint gives rise to a relation among various parameters

$$\nu\pi_4 = k\pi_0 - \omega\pi_2. \quad (2.6)$$

The second one becomes

$$\begin{aligned} 0 &= -\frac{1}{(b^2 - a^2)^2} (b^2 k^2 + 2bk\pi_0) \\ &\quad + \rho'^2 + \frac{1}{(b^2 - a^2)^2} \left(2b\omega\pi_2 + b^2(\omega^2 - k^2) \sinh^2 \rho + \frac{(\pi_2^2 - \pi_0^2) \sinh^2 \rho + \pi_2^2}{\cosh^2 \rho \sinh^2 \rho} \right) \\ &\quad + \theta'^2 + \frac{1}{(b^2 - a^2)^2} \left(\nu^2 b^2 \sin^2 \theta + 2b\nu\pi_4 + \frac{\pi_4^2}{\sin^2 \theta} \right). \\ &= -k_0^2 + k_1^2 + k_s^2, \end{aligned} \quad (2.7)$$

where in the last equality we have defined

$$\begin{aligned} k_0^2 &= \frac{1}{(b^2 - a^2)^2} (b^2 k^2 + 2bk\pi_0), \\ k_1^2 &= \rho'^2 + \frac{1}{(b^2 - a^2)^2} \left(2b\omega\pi_2 + b^2(\omega^2 - k^2) \sinh^2 \rho + \frac{(\pi_2^2 - \pi_0^2) \sinh^2 \rho + \pi_2^2}{\cosh^2 \rho \sinh^2 \rho} \right), \\ k_s^2 &= \theta'^2 + \frac{1}{(b^2 - a^2)^2} \left(\nu^2 b^2 \sin^2 \theta + 2b\nu\pi_4 + \frac{\pi_4^2}{\sin^2 \theta} \right). \end{aligned} \quad (2.8)$$

Then, the differential equations for ρ and θ can be rewritten as

$$\begin{aligned} \rho'^2 &= \frac{1}{(b^2 - a^2) \cosh \rho \sinh \rho} \left[b^2(\omega^2 - k^2) \sinh^6 \rho + \left(2b\omega\pi_2 + b^2(\omega^2 - k^2) \right. \right. \\ &\quad \left. \left. + (b^2 - a^2)k_1^2 \right) \sinh^4 \rho + \left(2b\omega\pi_2 + (b^2 - a^2)^2 k_1^2 + (\pi_2^2 - \pi_0^2) \right) \sinh^2 \rho + \pi_2^2 \right], \\ \theta'^2 &= \frac{1}{(b^2 - a^2)^2 \sin^2 \theta} \left[-\sin^4 \theta + \left(\frac{(b^2 - a^2)^2 k_s^2}{b^2 \nu^2} - \frac{2\pi_4}{b\nu} \right) \sin^2 \theta - \frac{\pi_4^2}{b^2 \nu^2} \right]. \end{aligned} \quad (2.9)$$

The equation for θ can be rewritten as

$$\theta' = \frac{b\nu}{(b^2 - a^2) \sin \theta} \sqrt{(\sin^2 \theta_{\max} - \sin^2 \theta)(\sin^2 \theta - \sin^2 \theta_{\min})}, \quad (2.10)$$

for $k_s^2(b^2 - a^2)^2 > 2b\nu\pi_4$ only. The condition $\sin \theta_{\max} = 1$ for the infinite size magnon or spike gives a relation

$$k_s = \frac{b\nu + \pi_4}{b^2 - a^2}. \tag{2.11}$$

Using this, $\sin \theta_{\min}$ becomes

$$\sin \theta_{\min} = \frac{\pi_4}{b\nu}, \tag{2.12}$$

where $b\nu > \pi_4$.

3 Classification of the string configurations on $AdS_3 \times S^2$

From now on, we will consider the special case, $k = \omega > 0$ only. An example for $k \neq \omega$ was investigated in ref. [30]. For $\kappa = \omega$, the equations of the AdS part are reduced to

$$\begin{aligned} \rho' &= \frac{1}{(b^2 - a^2) \cosh \rho \sinh \rho} \sqrt{\Delta \sinh^4 \rho + \Gamma \sinh^2 \rho - \pi_2^2}, \\ \phi' &= \frac{1}{(b^2 - a^2)} \left(a\omega - \frac{\pi_2}{\sinh^2 \rho} \right), \end{aligned} \tag{3.1}$$

with

$$\begin{aligned} \Gamma &= \Delta + \pi_0^2 - \pi_2^2, \\ \Delta &= (a^2 - b^2)^2 k_1^2 - 2b\omega\pi_2. \end{aligned} \tag{3.2}$$

Since there exists a solution only when the inside of the square root in the first equation of eq. (3.1) is positive, we will investigate the range of ρ accordingly. Depending on the parameters, the range of ρ can be classified as the follows:

$\Delta > 0$. In this case, there exists one boundary value ρ_{\min} so that the range of ρ is given by $\rho_{\min} \leq \rho < \rho_{\max} = \infty$ and ρ' is rewritten as

$$\rho' = \frac{\sqrt{\Delta}}{b^2 - a^2} \frac{\sqrt{\sinh^2 \rho + \sinh^2 \rho_0}}{\sinh \rho \cosh \rho} \sqrt{\sinh^2 \rho - \sinh^2 \rho_{\min}}, \tag{3.3}$$

where depending on the sign of Γ , one has the following choices

(i) $\Gamma > 0$

$$\begin{aligned} \sinh^2 \rho_0 &= \frac{\sqrt{\Gamma^2 + 4\pi_2^2 \Delta} + \Gamma}{2\Delta} \\ \sinh^2 \rho_{\min} &= \frac{\sqrt{\Gamma^2 + 4\pi_2^2 \Delta} - \Gamma}{2\Delta}, \end{aligned} \tag{3.4}$$

(ii) $\Gamma = 0$

$$\sinh^2 \rho_0 = \sinh^2 \rho_{\min} = \frac{\sqrt{\Delta + \pi_0^2}}{\sqrt{\Delta}}, \tag{3.5}$$

(iii) $\Gamma < 0$

$$\begin{aligned} \sinh^2 \rho_0 &= \frac{\sqrt{|\Gamma|^2 + 4\pi_2^2 \Delta} - |\Gamma|}{2\Delta} \\ \sinh^2 \rho_{\min} &= \frac{\sqrt{|\Gamma|^2 + 4\pi_2^2 \Delta} + |\Gamma|}{2\Delta}. \end{aligned} \tag{3.6}$$

In addition, the string configuration can also be classified into various cases by looking at the shape of the string. For this, we define the slope of the string at a fixed τ as

$$\mathcal{R} \equiv \left| \frac{\partial \rho}{\partial \phi} \right| = \left| \frac{\rho'}{\phi'} \right|. \tag{3.7}$$

For example, the string slope is infinity, and the string configuration has a cusp at ρ_{\min} . For more details, we introduce $\sinh^2 \rho_c = \frac{\pi_2^2}{a^2}$ satisfying $\phi' = 0$ in which the slope \mathcal{R} diverges.

- (i) If $\rho_c < \rho_{\min}$, the string slope becomes 0 at ρ_{\min} and a constant at $\rho_{\max} = \infty$.
- (ii) In the case of $\rho_c = \rho_{\min}$, as previously mentioned, the slope at ρ_{\min} becomes infinity so that this string configuration has a cusp at ρ_{\min} and a constant slope at $\rho_{\max} = \infty$.
- (iii) If $\rho_c > \rho_{\min}$ but finite, the string configuration is same as case (i) except that it includes a point ρ_c where the sign of the slope is opposite.
- (iv) If $\rho_c = \rho_{\max} = \infty$, the slope becomes zero at ρ_{\min} and infinity at ρ_{\max} . This case can be obtained when we consider $a = 0$.

$\Delta = 0$. In this case, the range of ρ is given by $\rho_{\min} \leq \rho < \infty$. For $\Gamma > 0$ ρ' becomes

$$\rho' = \frac{\sqrt{\Gamma}}{b^2 - a^2} \frac{\sqrt{\sinh^2 \rho - \sinh^2 \rho_{\min}}}{\sinh \rho \cosh \rho}, \tag{3.8}$$

with

$$\sinh^2 \rho_{\min} = \frac{\pi_0^2}{\pi_0^2 - \pi_2^2}. \tag{3.9}$$

The classification of the string configuration is similar to the previous case. Note that since ρ' becomes zero at $\rho = \infty$, the string configuration is slightly different from the previous cases. For $\rho_c < \rho_{\min}$, the slope \mathcal{R} vanishes at ρ_{\min} and $\rho_{\max} = \infty$. For $\rho_c = \rho_{\min}$, the string slope becomes infinity at ρ_{\min} and zero at $\rho_{\max} = \infty$. For $\rho_c > \rho_{\min}$ but finite, \mathcal{R} becomes zero at both ρ_{\min} and $\rho_{\max} = \infty$. For $\rho_c = \rho_{\max}$, with $a = 0$, \mathcal{R} is zero at ρ_{\min} and infinity at $\rho_{\max} = \infty$.

In the case of $\Gamma \leq 0$, there is no string configuration since the inside of the square root in the first equation of eq. (3.1) becomes negative and hence ρ' becomes imaginary.

$\Delta < 0$. In this case, there exist two boundary values for $\Gamma > 0$ so that the range of ρ is given by $\rho_{\min} \leq \rho \leq \rho_{\max}$. Then, ρ' is

$$\rho' = \frac{\sqrt{|\Delta|}}{b^2 - a^2} \frac{\sqrt{(\sinh^2 \rho_{\max} - \sinh^2 \rho) (\sinh^2 \rho - \sinh^2 \rho_{\min})}}{\sinh \rho \cosh \rho}, \quad (3.10)$$

where $\sinh^2 \rho_{\max}$ and $\sinh^2 \rho_{\min}$ are

$$\begin{aligned} \sinh^2 \rho_{\max} &= \frac{\Gamma + \sqrt{\Gamma^2 - 4\pi_2^2 |\Delta|}}{2|\Delta|}, \\ \sinh^2 \rho_{\min} &= \frac{\Gamma - \sqrt{\Gamma^2 - 4\pi_2^2 |\Delta|}}{2|\Delta|}. \end{aligned} \quad (3.11)$$

For $\rho_c < \rho_{\min}$ or $\rho_c > \rho_{\max}$, the slope vanishes at ρ_{\min} and ρ_{\max} . Unlike the previous cases, the case of $a = 0$ corresponds to the case of $\rho_c > \rho_{\max}$. For $\rho_c = \rho_{\min}$, the string configuration has a cusp at ρ_{\min} where \mathcal{R} becomes infinite and a zero slope at ρ_{\max} . For $\rho_{\min} < \rho_c < \rho_{\max}$, the slopes at ρ_{\min} and ρ_{\max} vanish and there exist a point where the sign of slope is changed. For $\rho_c = \rho_{\max}$, the slope vanishes at ρ_{\min} and becomes infinite at ρ_{\max} . Once again for $\Gamma \leq 0$, there is no string configuration because ρ' becomes imaginary.

In the infinite size limit for the giant magnon or spike solutions for the string on S^2 with $\theta_{\max} = \pi/2$, the conserved charges are given by:

$$\begin{aligned} E &= \left| \frac{2T}{(b^2 - a^2)b} \int_{\rho_{\min}}^{\rho_{\max}} d\rho \frac{(b^2 \omega \cosh^2 \rho - a\pi_0)}{\rho'} \right|, \\ S &= \left| \frac{2T}{(b^2 - a^2)b} \int_{\rho_{\min}}^{\rho_{\max}} d\rho \frac{(b^2 \omega \sinh^2 \rho - a\pi_2)}{\rho'} \right|, \\ J &= \left| \frac{2T}{(b^2 - a^2)b} \int_{\theta_{\min}}^{\pi/2} d\theta \frac{(b^2 \nu \sin^2 \theta - a\pi_4)}{\theta'} \right|, \\ \Delta\phi &= \left| \frac{2}{(b^2 - a^2)} \int_{\rho_{\min}}^{\rho_{\max}} d\rho \frac{1}{\rho'} \left(a\omega - \frac{\pi_2}{\sinh^2 \rho} \right) \right|, \\ \Delta\psi &= \left| \frac{2}{(b^2 - a^2)} \int_{\theta_{\min}}^{\pi/2} d\theta \frac{1}{\theta'} \left(a\nu - \frac{\pi_4}{\sin^2 \theta} \right) \right|, \end{aligned} \quad (3.12)$$

where we consider only the positive quantities. For simplicity, we assume that the ends of the open string are located at ρ_{\max} in the AdS space and at the same time, θ_{\max} on S^2 .

3.1 Spike on AdS_3 and a point-like string on S^2

At first, we consider the string configuration located at a point on S^2 ($\theta' = 0$). For this, we choose $\nu = \pi_4 = 0$, which makes the angular momentum and the angle difference in the ψ directions to vanish. So, the string solution for $\nu = \pi_4 = 0$ corresponds to the string extend in AdS but a static point particle on S^2 . Once again we consider the situation case by case like the previous section for different parameter regions.

$\Delta > 0$.

1. We first consider the case with $a \neq 0$ and $b^2\omega \neq a(\pi_0 - \pi_2)$. The interesting physical quantities of this string configuration are

$$\begin{aligned}
 E - S &= \frac{2(b^2\omega - a(\pi_0 - \pi_2))T}{b\sqrt{\Delta}} \\
 &\quad \times \int_{\rho_{\min}}^{\rho_{\max}} d\rho \frac{\sinh \rho \cosh \rho}{\sqrt{(\sinh^2 \rho + \sinh^2 \rho_0)(\sinh^2 \rho - \sinh^2 \rho_{\min})}}, \\
 S &= \frac{2T}{b\sqrt{\Delta}} \int_{\rho_{\min}}^{\rho_{\max}} d\rho \frac{\sinh \rho \cosh \rho (b^2\omega \sinh^2 \rho - a\pi_2)}{\sqrt{(\sinh^2 \rho + \sinh^2 \rho_0)(\sinh^2 \rho - \sinh^2 \rho_{\min})}}, \\
 \Delta\phi &= \frac{2}{\sqrt{\Delta}} \int_{\rho_{\min}}^{\rho_{\max}} d\rho \frac{\cosh \rho (a\omega \sinh^2 \rho - \pi_2)}{\sinh \rho \sqrt{(\sinh^2 \rho + \sinh^2 \rho_0)(\sinh^2 \rho - \sinh^2 \rho_{\min})}}, \quad (3.13)
 \end{aligned}$$

where $\rho_{\max} = \infty$. In the case of $\sinh \rho_{\min} \gg 1$, the dispersion relation can be approximately written as

$$E - S \approx \frac{b^2\omega - a(\pi_0 - \pi_2)}{ab\omega} T \Delta\phi. \quad (3.14)$$

Note that the dominant contribution of the above integral comes from the large ρ region. So the calculation of the integral gives rise to

$$\begin{aligned}
 E - S &\approx \frac{2(b^2\omega - a(\pi_0 - \pi_2))T}{b\sqrt{\Delta}} \rho_{\max}, \\
 S &\approx \frac{b\omega T}{4\sqrt{\Delta}} e^{2\rho_{\max}}. \quad (3.15)
 \end{aligned}$$

As a result, the dispersion relation for $\rho_{\max} = \infty$ becomes

$$E - S \approx \frac{(b^2\omega - a(\pi_0 - \pi_2))T}{b\sqrt{\Delta}} \log \left(\frac{4\sqrt{\Delta}}{b\omega T} S \right), \quad (3.16)$$

which is the generalization of the GKP string configuration [35].

2. Let us consider the case $b^2\omega = a(\pi_0 - \pi_2)$, the dispersion relation is

$$E - S = 0, \quad (3.17)$$

which looks like that of the BPS vacuum state. In this case, S and $\Delta\phi$ are given by

$$\begin{aligned}
 S &\approx \frac{b\omega T}{4\sqrt{\Delta}} \left[e^{2\rho_{\max}} + 4 \left(\sinh^2 \rho_{\min} - \sinh^2 \rho_0 - \frac{2a\pi_2}{b^2\omega} \right) \rho_{\max} \right], \\
 \Delta\phi &\approx \frac{2a\omega}{\sqrt{\Delta}} \rho_{\max}. \quad (3.18)
 \end{aligned}$$

So S and E can be rewritten in terms of $\Delta\phi$ as

$$E = S \approx \frac{b\omega T}{4\sqrt{\Delta}} \left[e^{\Delta\phi\sqrt{\Delta}/a\omega} - \left(e^{2\rho_0} - e^{2\rho_{\min}} + \frac{2a\pi_2}{b^2\omega} \right) \frac{\Delta\phi\sqrt{\Delta}}{2a\omega} \right]. \quad (3.19)$$

3. Now, consider the case $a = 0$. In this case, $\Delta\phi$ has a finite value because the integral is regular even at ρ_{\min} and $\rho_{\max} = \infty$. On the contrary, $E - S$ and S diverge at $\rho_{\max} = \infty$ so that we can not represent $E - S$ and S in terms of $\Delta\phi$. However, we can compute the dispersion relation for this string configuration, which is

$$E - S \approx \frac{b\omega T}{\sqrt{\Delta}} \log \left(\frac{4\sqrt{\Delta}}{b\omega T} S \right). \quad (3.20)$$

Notice the logarithmic behavior of the anomalous dimension.

$\Delta = 0$. For this case, $E - S$ and S are given by

$$\begin{aligned} E - S &\approx \frac{(b^2\omega - a(\pi_0 - \pi_2)) T}{b\sqrt{\Gamma}} e^{\rho_{\max}}, \\ S &\approx \frac{b\omega T}{12\sqrt{\Gamma}} e^{3\rho_{\max}}, \end{aligned} \quad (3.21)$$

so that the dispersion relation becomes

$$E - S \approx \frac{(b^2\omega - a(\pi_0 - \pi_2)) T}{b\sqrt{\Gamma}} \left(\frac{12\sqrt{\Gamma}}{b\omega T} \right)^{1/3} S^{1/3}. \quad (3.22)$$

Like the previous case, if we put $b^2\omega - a(\pi_0 - \pi_2) = 0$, the string configuration becomes BPS-like configuration. In this case, the dispersion relation becomes

$$E - S \approx 12^{1/3} \left(\frac{b\omega T}{\sqrt{\Gamma}} \right)^{2/3} S^{1/3}. \quad (3.23)$$

$\Delta < 0$. For this case, the conserved charges are given by

$$\begin{aligned} E - S &= \frac{(b^2\omega - a(\pi_0 - \pi_2)) T}{b\sqrt{|\Delta|}} \pi, \\ S &= \frac{T}{b\sqrt{|\Delta|}} \left(\frac{b^2\omega}{2} (\sinh^2 \rho_{\max} + \sinh^2 \rho_{\min}) - a\pi_2 \right) \pi. \end{aligned} \quad (3.24)$$

From these, the dispersion relation can be rewritten as

$$E - S = \frac{2(b^2\omega - a(\pi_0 - \pi_2)) |\Delta|}{b^2\omega\Gamma - 2a\pi_2 |\Delta|} S. \quad (3.25)$$

For $a = 0$, the dispersion relation is reduced to a simple form

$$E - S = \frac{2|\Delta|}{\Gamma} S. \quad (3.26)$$

3.2 Spike on AdS_3 and circular string on S^2

Here, we consider the following parameter region: $\nu = 0$ and $\pi_4 \neq 0$. As previously mentioned, $\nu = 0$ implies $\theta' = 0$. So the string configuration in this parameter region describes a circular string on S^2 , which is extended in the ψ -direction with an angular momentum J at the fixed $\theta = \theta_c$. To describe the angular momentum J and the angle difference $\Delta\psi$, θ is not a good variable so that using

$$\frac{d\theta}{d\rho} = \frac{\theta'}{\rho'}, \quad (3.27)$$

we can change the integral with respect to θ in eq. (3.12) to

$$\begin{aligned} J &= \left| -\frac{2T}{(b^2 - a^2)b} \int_{\rho_{\min}}^{\rho_{\max}} d\rho \frac{a\pi_4}{\rho'} \right|, \\ \Delta\psi &= \left| -\frac{2}{(b^2 - a^2)} \int_{\rho_{\min}}^{\rho_{\max}} d\rho \frac{\pi_4}{\rho' \sin^2 \theta_c} \right|. \end{aligned} \quad (3.28)$$

From these, we find

$$J = \frac{a \sin^2 \theta_c}{b} T \Delta\psi. \quad (3.29)$$

Especially, since the angular momentum becomes zero for $a = 0$, the string solution is reduced to a static circular string on S^2 .

For $\Delta > 0$, the dispersion relation is slightly modified to

$$E - S - J \approx \frac{(b^2\omega - a(\pi_0 - \pi_2 + \pi_4)) T}{b\sqrt{\Delta}} \log \left(\frac{4\sqrt{\Delta}}{b\omega T} S \right). \quad (3.30)$$

So the BPS-like configuration appears at $b^2\omega = a(\pi_0 - \pi_2 + \pi_4)$. The dispersion relation for $a = 0$ can be represented in terms of the angle difference $\Delta\psi$ on S^2 instead of $\Delta\phi$ on AdS

$$\begin{aligned} E - S &= \frac{b\omega \sin^2 \theta_c}{\pi_4} T \Delta\psi \\ &\approx \frac{b\omega T}{\sqrt{\Delta}} \log \left(\frac{4\sqrt{\Delta}}{b\omega T} S \right). \end{aligned} \quad (3.31)$$

For $\Delta = 0$, when $a \neq 0$ the modified dispersion relation is

$$E - S - J \approx \frac{(b^2\omega - a(\pi_0 - \pi_2 + \pi_4)) T}{b\sqrt{\Gamma}} \left(\frac{12\sqrt{\Gamma}}{b\omega T} \right)^{1/3} S^{1/3}, \quad (3.32)$$

and at $a = 0$ it becomes

$$E - S \approx 12^{1/3} \left(\frac{b\omega T}{\sqrt{\Gamma}} \right)^{2/3} S^{1/3} \quad (3.33)$$

In the case of $\Delta < 0$, the dispersion relation for $a \neq 0$ is modified to

$$E - S - J = \frac{2(b^2\omega - a(\pi_0 - \pi_2 + \pi_4)) |\Delta|}{b^2\omega\Gamma - 2a\pi_2 |\Delta|} S, \quad (3.34)$$

and one for $a = 0$ becomes

$$E - S = \frac{2|\Delta|}{\Gamma} S. \quad (3.35)$$

As shown in the above results, at $a = 0$ the dispersion relations for a circular string on S^2 are same as ones for a point-like string on S^2 .

3.3 Circular string on AdS_3 and magnon on S^2

In this section, we consider the string configuration located at $\rho = \rho_c$. Especially, for $\rho_c = \rho_{\min} = 0$ and $\pi_2 = 0$ the string configuration on $AdS_3 \times S^2$ reduces to the magnon or spike on $R \times S^2$ located at the center of AdS_3 .

For $\rho_c \neq 0$, the string configuration on the AdS_3 part describes a circular string wounded in the ϕ -direction. The dispersion relation for this configuration is given by

$$E - S - J = \frac{2T}{(b^2 - a^2)b} \int_{\theta_{\min}}^{\pi/2} d\theta \frac{b^2\omega - a(\pi_0 - \pi_2 - \pi_4) - b^2\nu \sin^2 \theta}{\theta'}, \quad (3.36)$$

where eq. (3.27) is used. To obtain magnon like solution, we impose

$$\begin{aligned} b^2\nu &= b^2\omega - a(\pi_0 - \pi_2 - \pi_4), \\ a\nu &= \pi_4, \end{aligned} \quad (3.37)$$

such that the values of $E - S - J$ and $\Delta\psi$ are finite. Using eq. (2.6), one gets

$$\left(b^2 - \frac{a^2\nu^2}{\omega^2}\right) \omega = (b^2 - a^2)\nu. \quad (3.38)$$

In this case, the dispersion relation is given by a similar form of a magnon on $R \times S^2$

$$E - S - J = 2T \sin \frac{p}{2}, \quad (3.39)$$

where $p = \Delta\psi$. In more general cases, the conserved charges are represented in terms of the Jacobi elliptic integrals so that it is not easy to find the explicit form of the dispersion relation. Note that the right hand side of the dispersion relation in eq. (3.36) does not depend on the value of ρ_c .

4 More general solution

In the previous sections, we have considered a solitonic string configuration (spike or magnon) on either AdS_3 or S^2 . In this section, we investigate more general solitonic string configuration expanding on both AdS_3 and S^2 space. Using eq. (3.27), the anomalous dimension for this string configuration can be rewritten as

$$\begin{aligned} E - S - J &= \frac{2T}{b(b^2 - a^2)} \left[\int_{\rho_{\min}}^{\rho_{\max}} d\rho \frac{b^2\omega - a(\pi_0 - \pi_2 - \pi_4) - b^2\nu}{\rho'} + \int_{\theta_{\min}}^{\pi/2} d\theta \frac{b^2\nu \cos^2 \theta}{\theta'} \right] \\ &= \frac{2(b^2\omega - a(\pi_0 - \pi_2 + \pi_4) - b^2\nu)T}{b(b^2 - a^2)} \int_{\rho_{\min}}^{\rho_{\max}} \frac{d\rho}{\rho'} + 2T \cos \theta_{\min}. \end{aligned} \quad (4.1)$$

The anomalous dimension is reduced to

(i) for $\Delta > 0$

$$E - S - J \approx \frac{(b^2\omega - a(\pi_0 - \pi_2 - \pi_4) - b^2\nu) T}{b\sqrt{\Delta}} \log \left(\frac{4\sqrt{\Delta}}{b\omega T} S \right) + 2T \cos \theta_{\min}, \quad (4.2)$$

(ii) for $\Delta = 0$

$$E - S - J \approx \frac{(b^2\omega - a(\pi_0 - \pi_2 - \pi_4) - b^2\nu) T}{b\sqrt{\Gamma}} \left(\frac{12\sqrt{\Gamma}}{b\omega T} \right)^{1/3} S^{1/3} + 2T \cos \theta_{\min}, \quad (4.3)$$

and

(iii) for $\Delta < 0$

$$E - S - J \approx \frac{2(b^2\omega - a(\pi_0 - \pi_2 - \pi_4) - b^2\nu) |\Delta|}{b^2\omega\Gamma - 2a\pi_2 |\Delta|} S + 2T \cos \theta_{\min}. \quad (4.4)$$

Especially, for $a\nu = \pi_4$ the string configuration on S^2 becomes a magnon, which has an infinite angular momentum J and a finite angle difference $\Delta\psi$. In this case, the worldsheet momentum p equal to the angle difference $\Delta\psi$ is given by

$$\cos \frac{p}{2} = \sin \theta_{\min}. \quad (4.5)$$

Using this result, the anomalous dimension is

$$E - S - J = \frac{2(b^2\omega - a(\pi_0 - \pi_2) - (b^2 - a^2)\nu) T}{b(b^2 - a^2)} \int_{\rho_{\min}}^{\rho_{\max}} \frac{d\rho}{\rho'} + 2T \sin \frac{p}{2}, \quad (4.6)$$

where the integral with respect to ρ is proportional to

(i) $\log S$ for $\Delta > 0$,

(ii) $S^{1/3}$ for $\Delta = 0$ and

(iii) S for $\Delta < 0$.

If $b^2\omega = a(\pi_0 - \pi_2) + (b^2 - a^2)\nu$, the dispersion relation for this configuration becomes to that of the magnon on S^2

$$E - S - J = 2T \sin \frac{p}{2}. \quad (4.7)$$

Though the above dispersion relation is the same as that for the circular string located at $\rho = \rho_c$ in the section 3, the string configuration considered here is a more general one extended in the ρ direction.

When $a\pi_4 = b^2\nu$, the string configuration on S^2 becomes spike, which has a finite angular momentum and an infinite angle difference. The difference of the conserved charges becomes

$$E - S - J = \frac{2(b^2\omega - a(\pi_0 - \pi_2)) T}{b(b^2 - a^2)} \int_{\rho_{\min}}^{\rho_{\max}} \frac{d\rho}{\rho'} + 2T \cos \theta_{\min}. \quad (4.8)$$

The dispersion relation for the spike configuration is given by

$$E - S - T\Delta\psi = \frac{2(b^2\omega - a(\pi_0 - \pi_2) + b(b^2 - a^2)\nu/a) T}{b(b^2 - a^2)} \int_{\rho_{\min}}^{\rho_{\max}} \frac{d\rho}{\rho'} + 2T\tilde{\theta} \quad (4.9)$$

where $\tilde{\theta} = \pi/2 - \theta_{\min}$ and the integral with respect to ρ splits to the three cases like eq. (4.2), eq. (4.3) and eq. (4.4) depending on the value of Δ . For $a(\pi_0 - \pi_2) = b^2\omega + b(b^2 - a^2)\nu/a$, this dispersion relation looks like one for a spike on S^2 .

5 Conclusions

In this paper, we have investigated a class of string configurations on $AdS_3 \times S^2$ with a particular parameter relation $k = \omega$.

For $k = \omega$, there are three kinds of the string configuration on the AdS_3 space. Each of these configurations has a different anomalous dimension proportional to $\log S$ for $\Delta > 0$ and $S^{1/3}$ for $\Delta = 0$. In these cases, the string is extended from the minimum of ρ to infinity and depending on the position of ρ_c where the string slope \mathcal{R} diverges, the string configurations can be split into several cases by studying the shape of them. For $\Delta < 0$ since there exist two extremum points corresponding to minimum and maximum of ρ , the range of ρ is given by $\rho_{\min} \leq \rho \leq \rho_{\max}$, in which the anomalous dimension is proportional to S .

Furthermore, these string configurations have been generalized to the $AdS_3 \times S^2$ case having two angular momenta, S in the AdS and J in the S^2 . Here, we have obtained the generalized dispersion relation for the string configuration on $AdS_3 \times S^2$.

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